EXTENDED MAXWELL PAIR GRADIENT COILS

FIELD OF THE INVENTION

This invention is in the technical field of coil design, for example, for use in nuclear magnetic resonance (NMR) applications.

BACKGROUND OF THE INVENTION

This invention relates to means for providing a magnetic field with a uniform gradient suitable for use in NMR applications. A classic approach to this purpose has been to use a pair of loop coils known as the Maxwell pair, characterized as being placed symmetrically about the sample, carrying equal but opposite currents and being separated by a distance $\sqrt{3}$ times the loop radius.

There have been at least the following two problems with a Maxwell pair. One of them is that the gradient coils of a Maxwell pair is usually constructed with an additional coil wound at a larger radius and carrying a current so as to cancel the magnetic field exterior to the coil structure and the presence of such shield windings affects the condition for linearity of the gradient of the magnetic field being generated. The other of the problems is that a Maxwell pair thus shielded can be constructed as long as the diameter of the wire used for the pair is small enough compared to the radius of the coil, but this makes the gradient strength per ampere of current too small for NMR applications. If additional windings are to be added in order to enhance the field gradient, however, the spatial extent of these windings must also be small compared to the loop diameter of the pair.

SUMMARY OF THE INVENTION

It is therefore an object of this invention to provide a pair of extended coils, such that currents may reside along a length comparable to the loop diameter, capable of providing a magnetic field with a stronger uniform field gradient.

An extended Maxwell pair embodying this invention, with which the above and other objects can be accomplished, may be characterized as comprising a pair of cylindrical gradient coils disposed coaxially and adapted to carry equal currents in mutually opposite directions. Each of these gradient coils may be surrounded by a coaxially disposed cylindrically extended shield coil for canceling magnetic fields outside. For given values of radii of the gradient and shield coils, the length and the center-to-center separation of the pair of gradient coils are

determined by numerically solving an equation which is derived from the condition that the currents through the gradient and shield coils should together generate a magnetic field inside with a linear gradient. The equation to be solved is derived by calculating the magnetic field by a Fourier-Bessel expansion method incorporating the condition that the shield coils do shield the magnetic field inside. The manner in which a wire should be wound to form the shield coils is determined from the numerical solution of the equation.

BRIEF DESCRIPTION OF THE DRAWINGS

The accompanying drawings, which are incorporated in and form a part of this specification, illustrate embodiments of the invention and, together with the description, serve to explain the principles of the invention. In the drawings:

Fig. 1 is a sketch of an extended Maxwell pair of gradient coils embodying this invention for showing their dimensional relationships;

Fig. 2 is a graph showing an example of current function related to the primary coils of Fig. 1 when they are formed by a uniformly and tightly wound wire;

Fig. 3 is a sketch of an example of primary coil;

Fig. 4 is a graph showing the current function related to primary coils structured as shown in Fig. 3; and

Fig. 5 is a graph of a portion of a current function related to the secondary coils which may be obtained by a method of this invention and for showing a method of winding a wire to form such secondary coils.

DETAILED DESCRIPTION OF THE INVENTION

This invention relates to a so-called extended Maxwell pair 10 comprising, as schematically shown in Fig. 1, a coaxially disposed pair of cylindrical primary (gradient) coils 11 and 12 and another coaxially disposed pair of cylindrical secondary (shield) coils 15 and 16. The radius and the length of each of the primary coils 11 and 12 are denoted by letters a and d, respectively, and their common central axis will be defined as the z-axis, for the convenience of description. The center-to-center separation between the two primary coils 11 and 12 is denoted by symbol z_0 (>d), and the point of symmetry on the z-axis is defined as the origin O of the coordinate system to be used for the description. The radius of the secondary coils 15 and 16 is denoted by letter b (>a). The shield coils 15 and 16 are disposed not only coaxially with respect

to each other but also with the primary coils 11 and 12, each enclosing a corresponding one of the primary coils 11 and 12 inside. Each of the shield coils 15 and 16 is at least of the length of the primary coils 11 and 12.

Although not shown in Fig. 1 for the convenience of disclosure, the two pairs of these coils 11, 12, 15 and 16 are adapted to be fed equal currents, the currents through each pair being in mutually opposite directions and the currents through each of the primary coils 11 and 12 and the corresponding one of the secondary coils being also in opposite directions.

A method of using a Fourier-Bessel expansion of the magnetic field generated by currents flowing on a cylinder has been discussed by R. Turner (in an article entitled "A target field approach to optimal coil design" which appeared in J. Phys. D: Appl. Phys. 19 (1986) L147-L151) and the problem of shielding a gradient magnetic field has been discussed by R. Turner and R. M. Bowley (in an article entitled "Passive screening of switched magnetic field gradients" which appeared in J. Phys. E: Sci. Instrum. 19, 876 (1986)). Both of these articles will be herein incorporated by reference.

Because the coils 11, 12, 15 and 16 are arranged in a cylindrically symmetric manner, cylindrical coordinates will be used, as done in the incorporated references which showed, given a surface current distribution $j(\varphi, z)$ flowing on the surface of a cylinder of radius a, how to write the axial component (z-component) of the magnetic field B_z in the form of Fourier-Bessel series. Where, as here, the configuration is symmetric, however, this series expression becomes much simplified.

For the convenience of discussion, let $j^P(\varphi, z)$ indicate the surface current distribution (or current density in units of ampere/cm) of the primary coils 11 and 12 and the current function $I^P(z)$ (in units of amperes) related to the primary coils 11 and 12 be defines as follows:

$$I^P(z) = \int_{-\infty}^{z} dz' j^P(\varphi, z').$$

If the primary coils 11 and 12 are each formed by tightly winding a thin wire uniformly a same number of times, for example, such that the azimuthal current density is nearly constant (a positive number for one of the coils and the same negative number for the other) over a distance d along the z-axis for each coil and zero elsewhere, the corresponding current function $I^P(z)$ will looked as shown in Fig. 2, the small steps in the sloped regions corresponding to the individual loops of the wound wire. If the primary coils 11 and 12 are each formed by helically rolling a rectangular conductor sheet, as shown in Fig. 3, the current therethrough will be uniform along the z-axis and the corresponding current function $I^P(z)$ will look as shown in Fig. 4, the small steps in Fig. 2 disappearing. If the surface current distribution of the primary coils 11 and 12 is

such that their current function is given as shown in Fig. 2, or approximately as shown in Fig. 4, the azimuthal component of their current distribution, Fourier-transformed into k-space, is given by:

$$J_\varphi^P(k) = 2i(I/d)\sin(kd/2)\sin(kz_0/2)/(kd/2) \quad (1)$$

where $i = \sqrt{-1}$. The corresponding current function $I^P(k)$ in k-space is related to this as follows:

$$J_\varphi^P(k) = kI^P(k).$$

The axial component of this distribution can be obtained therefrom by conservation of current. If the azimuthal component of the current distribution of the secondary coils 15 and 16 (each with radius b) is similarly Fourier-transformed into k-space, the condition that the secondary coils 15 and 16 serve to cancel the exterior field (hereinafter referred to as "the shielding condition") is translated into the following relationship:

$$J_\varphi^s(k) = -(a/b)(I_1(ka)/I_1(kb))J_\varphi^P(k) \quad (2)$$

where I_1 is the first order modified Bessel function of the first kind. In terms of the current functions for the primary and secondary coils, the aforementioned shielding condition is written as follows:

$$I^s(k) = -(a/b)(I_1(ka)/I_1(kb))I^P(k) \quad (2')$$

where $I^s(k)$ is defined in terms of the surface current density $j^s(\varphi, z)$ of the secondary coils as $I^P(k)$ was defined above in terms of $j^P(\varphi, z)$.

The z-component of the magnetic field B_z near the origin O can be calculated by a method described in the cited references and will take the following form, if Taylor-expanded into a polynomial form:

$$B_z(z) = B_z(0)+c_1z+c_3z^3+c_5z^5+\cdots$$

where $B_z(0)=0$ and c_1, c_3, c_5, etc. are constants not including z, the terms with even powers of z being excluded because of the symmetry property of the system.

To improve the linearity of the gradient near the origin O, the first non-linear term is set equal to zero, or $c_3=0$. This leads to the following linearity-establishing condition:

Sub a1

$$\int_0^{k_{max}} dk\, k^5 I_\varphi^P(k)S_0(k)K_0'(ka)I_0(k\rho)=0 \quad (3)$$

where ρ is the distance from the z-axis, K is the Bessel function of the second kind, k_{max} is a suitable upper limit of the integration, and $S_0(k)$ may be referred to as the shielding factor, given by

$$S_0(k)=1-K_1(kb)I_1(ka)/K_1(ka)I_1(kb). \quad (4)$$

The linearity-establishing condition (3) given above should ideally hold for all values of ρ. For practical applications in NMR, however, ρ may be set equal to the maximum value of a region

occupied by the sample. For an NMR tube of radius 5mm, for example, ρ may be set equal to 2.5mm.

If the primary coils 11 and 12 are structured such that their current distribution is given by (1), in particular, the linearity-establishing equation takes the following form:

$$\int_0^{k_{max}} dk k^4 \{\sin(kd/2)\sin(kz_0/2)/(kd/2)\} S_0(k) K_0'(ka) I_0(k\rho)=0.$$

For designing an extended Maxwell pair embodying this invention, the equation given above for the linearity-establishing condition is numerically solved to obtain z_0, once values for a, b and d are selected. It may alternatively be solved for d by selecting values of a, b and z_0.

After both the length d and the separation z_0 of the primary coils 11 and 12 have been thus determined, $I^s(k)$ becomes a function of a known form from the relationship (2) given above representing the shielding condition. The manner in which the secondary coils 15 and 16 should be wound may be determined as follows from $I^s(z)$ which is obtained by inverse Fourier transformation of $I^s(k)$ back into z-space.

The inverse Fourier-transformed function $I^s(z)$ is approximately of a similar functional form as that of $I^p(z)$ except that they are negative to each other, the currents flowing in opposite directions through the coils 11 and 15 and through the coils 12 and 16 with reference to Fig. 1. In other words, $I^s(z)=0$ at $z=-\infty$ and becomes negative at the left-hand end of the left-hand secondary coil 15, reaches a maximum negative value at the right-hand end of the left-hand secondary coil 15, remains at this maximum negative value until z is increased to the left-hand end of the right-hand secondary coil 16, and returning to zero at the right-hand end of the right-hand secondary coil 16. For the convenience of disclosure, Fig. 5 shows only a portion of the curve for $I^s(z)$ for $z<0$. Generally, $I^p(z)$ and $I^s(z)$ are of different functional forms, and this means that the secondary coils are not formed by winding a wire uniformly even if the primary coils each have a uniform current distribution as shown in Figs. 2-4.

If $I^s(z)$ is as shown in Fig. 5 and if each of the secondary coils 15 and 16 is to be formed by winding a wire N times where N is an integer, one of the methods would be to divide the distance along the vertical axis of Fig. 5 into N equal segments between the origin O and the position represented by $I^s(0)$, identifying the vertical coordinates of the middle points of these segments (indicated by symbols y_j in Fig. 5 where $j = 1, 2, \cdots, N$), noting the points on the curve having these vertical coordinates (indicated by symbols P_j in Fig. 5 where $j = 1, 2, \cdots, N$) and determining the z-coordinates (indicated by symbols z_j in Fig. 5 where $j = 1, 2, \cdots, N$) of these points P_j. The secondary coil 15 is formed by winding a wire at axial positions with the z-coordinates z_j thus determined.

Although the invention was described above with the assumption that the primary coils 11 and 12 are evenly wound, it should be clear from the description above that it is not a required condition. With a different functional form of $j^P(\varphi, z)$ and hence with that of $J_\varphi{}^P(k)$, the equation for the linearity-establishing condition requires a different functional form of $J_\varphi{}^s(k)$ from which a different current distribution $j_\varphi{}^s(z)$ for the secondary coils 15 and 16 will be obtained.

The present invention is further applicable to extended Maxwell pairs without secondary coils (shield coils). In the absence of the secondary coils, the shielding factor S_0 is set equal to 1 and the equation shown above for the linearity-establishing condition is numerically solved either for d, given a, b and z_0 or for z_0, given a, b and d.

The disclosure presented above is not intended to limit the scope of the invention. The disclosure is intended to be interpreted broadly. For example, each primary coil designed for a uniform current distribution over a distance of d along the z-axis may be formed by winding a thin wire tightly and uniformly to approximate such a uniform current distribution or by spirally rolling a conductor sheet of width d around the z-axis. The invention does not impose any limitation as to the relative magnitude of a and d. Of particular interest, however, are examples wherein a and d are of a same order of magnitude, in contrast to traditional Maxwell pairs for which $d \ll a$.